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FUNCTIONAL ANALYSIS AS AN EFFICIENT INSTRUMENT FOR SIGNALPROCESSING IN GEOTECHNICS

Abstract

Contribution works out with the problem for rocks disintegration process operation by rotary drilling, where objective function is to minimize specific energy of the rocks disintegration. Proposed operational algorithm is based on continuous evaluating of associated vibrations, on grounds of current disintegrated rock is classified to relevant class from classes, which are defined in advance. For each of classes are expertness specify parameters for effective drilling. Individual rocks are interpreted by associated vibrating signals from the drilling process as a vector in infinitely complex Hilbert's space. Solution is based on the knowledge of the functional analyses and on the methods of the artificial intelligence.

1 INTRODUCTION

A little group of people at Faculty of Mining, Ecology, Process Control and Geotechnologies of the Technical University Košice (FBERG) is concerning with rocks disintegration process operation by rotary drilling for a longer time. Scientific research in the field is intensively expanding at Institute of Geotechnics SAS in Košice where experimental drilling stand is presented. Both groups tightly cooperate in the field of research.

Group at FBERG is specializing in efficiency of drilling process operation based on digital signal processing of inherent vibro – acoustic signal including applying of several methods of artificial intelligence. A task described in the article was solved and processed as a component of scientific research grant program [1] (Leššo, I. et al., 2009). Content and used methods are relative to basic research.

Suggested solution for managing a rock solid drilling process is arising from concept of parametric drilling process operation, where after identifying a type of presently done rocks disintegration (identification by its vibro-acoustic exposition) is from database selected a mode of drilling, which is for the type of rock the most effective mode on the part of specific disintegration energy. An essence of the research is devoted to a task of identifying of a rock type, especially to a task of classification of rock into classes using methods based on inherent vibrations.

The solution of the classification of rocks, represented by the inherent vibration, is based on the relatively difficult functional analyses theory. Solvers are arising from some of theirs earlier

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works, eg. [2] (Leššo, I. et al., 2010). Functional analysis abstracts the concept of "space" and makes available a useful mathematical apparatus, which is similar in many moments to algebraic calculations, which are known in the classical three-dimensional Euclid space. Choosing the right type of space and the corresponding algebra gives some surprisingly powerful tool for solving complex tasks. This way of physical problems solution was presented by Einstein, Schrödinger and others in the past. The work is applying a Hilbert space, where rocks are placed as infinite dimensional vectors. At this point it is possible to put quotation of Prof. Naylor and Prof. Sella of work [3], where he writes, "*It is invaluable importance of Hilbert spaces for the natural and technical sciences*". The results presented in this paper also show negotiability of this procedure, based on strong mathematical abstraction.

2 ASSESSMENT OF LINEARITY OF THE SOLVED PROBLEM

The condition of applicability of the methods of linear algebra, where Hilbert spaces belong as linear spaces, is the linearity of the problem being solved. The linearity of the process under investigation means, among other things, the validity of the principle of superposition and the possibility of applying linear operators in the solution of the task.

The subject of investigation is the process of drilling a block of rock which, from the physical point of view, constitutes a heterogeneous continuum with infinite number of degrees of freedom. The concurrent vibration signal can be considered as a response of the rock to the polyharmonic stimulation by a rotating separating indenter. Through the process of discretization of continuum the block of rock was model-substituted with a linear discrete system with one fixed mass point, one degree of freedom, elastic clamping, damping, and external polyharmonic stimulation (Fig. 1). The solution of the motion equation of this oscillating system for polyharmonic stimulation was created on the basis of assumption of linearity of the system as a linear combination of particular solutions for individual harmonic components of the stimulating force. The solution is in the form

$$x(t) = \sum_{k=-\infty}^{\infty} x_{pk} = \sum_{k=-\infty}^{\infty} \hat{x}_{\max k} e^{i(k\Delta\omega t + \varphi_k')}, \quad (1)$$

where $\Delta\omega = \frac{2\pi}{T}$ is the frequency resolution determined by the period of the rotating indenter, φ_k' is

the phase of the k – component of the polyharmonic stimulation force. Formula (1) is a mathematical expression of the scanned vibration signal. It is obvious from its structure that the vibration signal in this case is substituted with a sum of theoretically infinite number of harmonic components with complex amplitudes. With a classical method of oscillation mechanics it was derived that these complex amplitudes are functions of mechanical properties of the oscillating continuum, so that

$$\hat{x}_{\max k} = \hat{x}_{\max k} \left(F_k, k, b, \frac{k\Delta\omega}{\omega_0} \right), \quad (2)$$

where F_k is the real amplitude of the k th component of the stimulating force, k is the elasticity and b is the damping of the continuum, ω_0 is its own frequency. From the physical point of view, the last three parameters are determined by geomechanical properties of the separated rock.

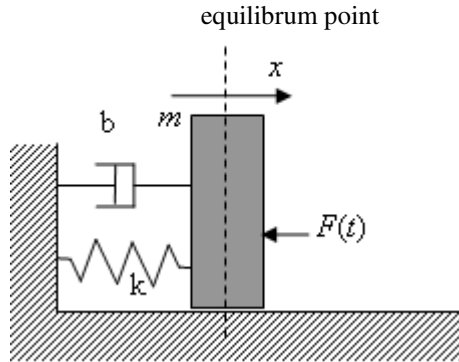


Fig.1: Model approach to analysis of concurrent vibrations in the process of drilling the block of rock.

In the following picture (Fig.2) are shown amplitude spectra of the vibrations from separating three different rocks using the same pressure and the same revolutions. From the point of view of model relation (1) it is about the magnitudes of complex amplitudes $|\hat{x}_{\max k}|, k = 0, 1, \dots, 1023$, obtained with the DTFT algorithm. The behavior of the spectra are in compliance with the structure of theoretically derived relations (1) and (2) for oscillation of the linear system stimulated by a periodic polyharmonic force. The spectra of all three rock contain components of the same frequencies (dential to frequencies of the components of the stimulating force), the difference being in the amplitudes of these components which are determined by geomechanical properties of rocks. This only confirms the substantiality of the approach to the task of investigating the process of rotary drilling of rock with linear methods and also representing the rocks with the aid of concurrent vibrations as functions in linear Hilbert space. At the same time Fig. 2 suggests differentiability of rocks relative to their own geomechanical parameters which in turn determine the effective mode of their drilling.

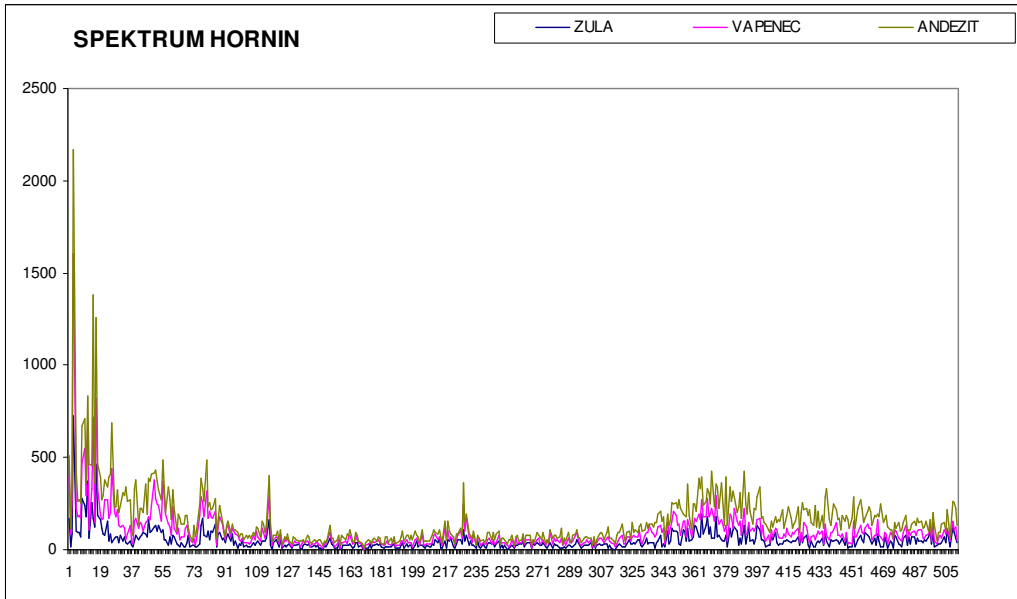


Fig.2: Amplitude spectra of concurrent vibrations from drilling three rocks (andesite, limestone, granite) at the same pressure and rpms.

3 A REPRESENTATION OF THE ROCKS AS VECTORS IN HILBERT SPACES AND ASSESSMENT OF THEIR MUTUAL DIVERGENCE

The goal of this part of the scientific research was to verify the appropriateness of rocks representation by its vibro signal from the drilling process. Were analyzed three types of rocks: andesite, limestone, granite, with drilling in the same mode (speed and downforce) in experimental drilling stand ÚGD SAS. Each rock was then represented by a single execution of vibro signal with length $n = 1024$ samples. This signal then represents the function (in theory, the continuous function) on the interval $\langle 0, T \rangle$, which can be seen as a vector, respectively point of infinitely large Hilbert spaces $H \equiv \square^\infty$. Then each rock r_i from n_R rocks represents infinite dimensional vector in Hilbert spaces

$$r_i = \mathbf{x}_i = (x_i(t), t: 0 \rightarrow T), \quad i=1,2,\dots,n_R. \quad (3)$$

Using topological and algebraic structure of Hilbert spaces can be defined scalar conjunction between two rocks as vectors

$$(\mathbf{x}_i, \mathbf{x}_j) = (x_i(t), x_j(t)) = \int_0^T x_i^*(t) x_j(t) dt, \quad (4)$$

and standard of vector as its length

$$\|\mathbf{x}_i\| = \left(\int_0^T |x_i(t)|^2 dt \right)^{1/2}, \quad (5)$$

and metrics generated by standard

$$\rho_p(\mathbf{x}_i, \mathbf{x}_j) = \|\mathbf{x}_i - \mathbf{x}_j\| = \left(\int_0^T |x_i(t) - x_j(t)|^2 dt \right)^{1/2}, \quad (6)$$

which determines the distance between two rocks in the Hilbert metric space.

Based on analogy with the scalar conjunction of two vectors in the classical Euclidean geometry is possible to use Cauchy - Schwarz inequality, which actually expresses the cosine of the angle between two vectors of Hilbert space

$$(\mathbf{x}_i, \mathbf{x}_j) \leq \|\mathbf{x}_i\| \|\mathbf{x}_j\|; \quad \int_0^T x_i^*(t) x_j(t) dt \leq \left(\int_0^T |x_i(t)|^2 dt \right)^{1/2} \left(\int_0^T |x_j(t)|^2 dt \right)^{1/2}. \quad (7)$$

The following figures Fig. 3 and Fig. 4 are showing the results for processing of the thirty vibro signal executions from each of the analyzed rocks. In Fig.3 in a so-called feature-based plane are showed the standard values of the particular rocks as vectors and standards of these rocks expressed with regard to the Fourier series in regard to an orthonormal base of harmonics functions (spectrum). At the same time are shown the centroids of clusters arisen from implementation of individual rocks. Sufficient differentiation in this standardized area is among andesite and two other rock ie. granite and limestone. Granite and limestone have small divergence, but they are also differentiable.

Fig. 4 is expressing an "angle" between the pairs of analyzed rocks as vectors in Hilbert space, which is based on the Schwarz inequality (correlation 7). Here was confirmed differentiability among andesite, granite and limestone.

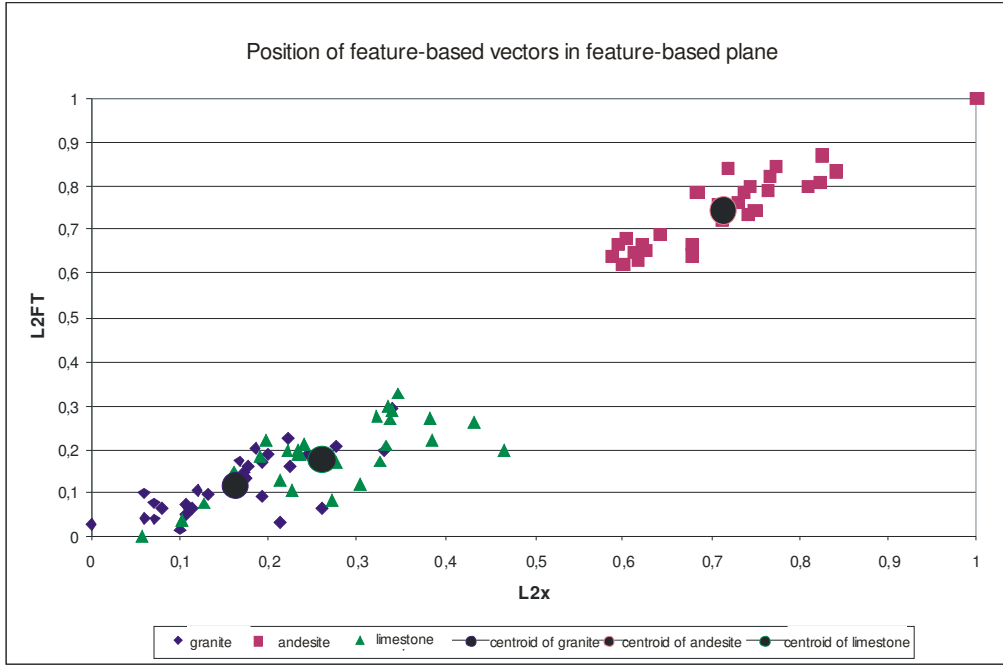


Fig.3: A dependency of symptoms $\|x\| - \|x\|$.

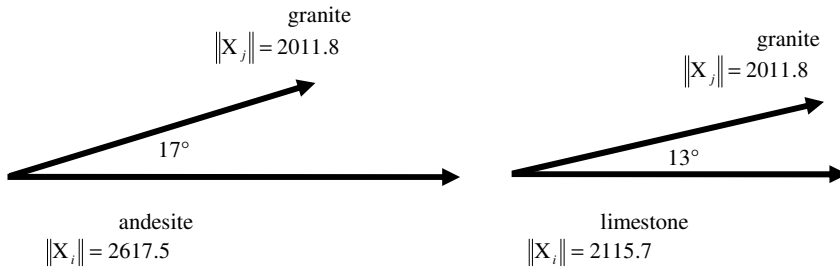


Fig.4: An angle between pair of rocks as vectors in Hilbert space H

4 CONCLUSION

In this paper are illustrated the partial results of scientific research on effective management of the rock disintegration process during rotary drilling. Drilling process control algorithm considers the classification of just disintegrated rock into rocks classes, to which are assigned speed and thrust by experts and where is reported minimum specific energy of disintegration [5, 6]. As a feature-based space of drilling process was used (based on the functional analysis) Hilbert space, where every rock is represented by its inherent vibro signal. The paper presents the basic algebraic and topological structure of such an infinite dimensional space and illustrates the differences in the position of the three types of rocks in that area. First results are confirming adequate differentiability among rocks or

among their inherent vibrations. Further research will be focused on confirmation of existing knowledge for a larger number of analyzed rocks. Research in this direction is necessary for the successful application of neural network as a classification of rocks by method known as vector quantization.

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